hw set 8
sect 5.8
p 410

1. $\int \frac{x^3-x^2+x-1}{x^2+9}\,dx$

$$
\begin{array}{r}
x-1 \\
x^2+0x+9 \,\overline{\big)\, x^3-x^2+x-1} \\
\underline{x^3+0x^2+9x} \\
-x^2-8x-1 \\
\underline{-x^2+0x+9} \\
-8x+8
\end{array}
$$

$= \int x-1+\frac{-8x+8}{x^2+9}\,dx$

$= \frac{1}{2}x^2 - x - \int \frac{8x}{x^2+9} + \int \frac{8}{x^2+9}\,dx$

$= \frac{1}{2}x^2 - x - 4\ln(x^2+9) + \frac{8}{3}\arctan\left(\frac{x}{3}\right) + C$

2. $\int e^{2\theta}\sin 3\theta\,d\theta$ use $\boxed{98}$ $\boxed{a=2,\ b=3}$

$= \frac{e^{2\theta}}{2^2+3^2}(2\sin 3\theta - 3\cos 3\theta)$

$= \frac{e^{2\theta}}{13}(2\sin 3\theta - 3\cos 3\theta) + C$

3. $\int \sec^3(\pi x)\,dx$ use $\boxed{71}$

$u = \pi x$

$du = \pi\,dx \Rightarrow dx = \frac{1}{\pi}du$

$= \int \sec^3 u\cdot\frac{1}{\pi}du = \frac{1}{\pi}\int \sec^3 u\,du$

$= \frac{1}{\pi}\left[\frac{1}{2}\sec u\tan u + \frac{1}{2}\ln(\sec u+\tan u) + C\right]$

$= \frac{1}{2\pi}\sec(\pi x)\tan(\pi x) + \frac{1}{2\pi}\ln|\sin(\pi x)+\tan(\pi x)|$

4. $\int_2^3 \frac{1}{x^2\sqrt{4x^2-7}}\,dx$ use $\boxed{45}$

$u^2 = 4x^2,\ u = 2x$

$du = 2\,dx$ and $x^2 = \frac{1}{4}u^2$

$= \int_4^6 \frac{1}{\frac{1}{4}u^2\sqrt{u^2-7}}\cdot\frac{1}{2}du$

$= 2\int \frac{1}{u^2\sqrt{u^2-(\sqrt{7})^2}}\,du$

$= 2\left[\frac{\sqrt{u^2-(\sqrt{7})^2}}{(\sqrt{7})^2 u} + C\right] = \frac{2\sqrt{u^2-7}}{7u}$

$= 2\frac{\sqrt{4x^2-7}}{7(2x)} = 2\frac{\sqrt{4x^2-7}}{14x}\Big|_2^3 =$

$= \cancel{2}\frac{\sqrt{4\cdot 9-7}}{21} - \cancel{2}\frac{\sqrt{16-7}}{14} = \frac{\sqrt{29}}{21} - \frac{\sqrt{9}}{14}$

$= \frac{\sqrt{29}}{21} - \frac{3}{14}$

$= \frac{2\sqrt{29}-9}{42}$

5. $\int \frac{dx}{x^2\sqrt{4x^2+9}}$ use $\boxed{28}$

$u^2 = 4x^2,\quad x^2 = \frac{1}{4}u^2$

$u = 2x \quad du = 2dx$

$= \int \frac{\frac{1}{2}du}{(\frac{1}{4}u^2)\sqrt{u^2+3^2}}$

$= 2\int \frac{du}{u^2\sqrt{u^2+3^2}}$

$= 2\left[-\frac{\sqrt{3^2+u^2}}{3^2 u}\right]$

$= \frac{-2\sqrt{u^2+9}}{9u} = \frac{-2\sqrt{4x^2+9}}{18x}$